Delayed Probabilistic Risk:
A Parametric Approach for Risk Tolerance

Jinrui Pan, Craig Webb & Horst Zank *

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Abstract: Experimental studies suggest that individuals exhibit more risk tolerance when prospects are delayed, leading to preference reversals that cannot be attributed to discounting. When data suggests that utility is time independent, probability weighting functions, such as those used to model prospect theory preferences, can account for such risk tolerance. We propose a descriptive model with a two-parameter probability weighting function where one of these parameters depends on the time at which a prospect is resolved. The time-dependent parameter is responsible for the curvature of the probability weighting function and can be seen as an index of insensitivity towards changes in probabilities. We provide conditions that characterize increased sensitivity towards more distant probabilities; this can account for the observed risk tolerance towards delayed prospects. In our framework, the discount function can be quite general and independent of utility for outcomes or probability weighting, allowing the model to be compatible with latest empirical findings of nonconstant discounting. In a simple application to bargaining we show that it can be advantageous for an individual to advance or delay the bargaining time if an opponent displays increased sensitivity to delay.

Keywords: bargaining, discounting, probability weighting, risk preferences, time preferences.

Journal of Economic Literature Classification Numbers: D81, D90.

*Jinrui Pan (jinrui.pan@durham.ac.uk): Business School, Durham University, UK; Craig S. Webb (craig.webb@manchester.ac.uk) & Horst Zank (horst.zank@manchester.ac.uk): Department of Economics, University of Manchester, UK. Correspondence: horst.zank@manchester.ac.uk.

1 Introduction

Many decisions taken by individuals concern risky outcomes obtained at specific points in time. These range from the purchase of lottery tickets, the ordering of goods that need to be delivered at certain dates, the booking of accommodation for business or holidays, to more complex financial instruments such as options or employment contracts with performance based pay components. Traditionally, such streams of risky outcomes are evaluated by taking a weighted average of the future expected utility (EU) of each risky option, where the weights are determined using a constant discount rate. The literature has questioned this form of discounting from a descriptive point of view (Loewenstein and Prelec 1992) as well as the use of EU for risk (Starmer 2000) and has called for more flexibility in modeling attitudes towards probabilities and temporal discounting.

In this paper we propose a descriptive discounted utility (DU) model, which separates temporal discounting from the utility of outcomes and also from the attitudes towards probabilities of outcomes. The latter, however, still depend on time as we adopt prospect theory (PT) type preferences for the evaluation of risky outcomes. Specifically, we extend the two-parameter inverse-S probability weighting function of Abdellaoui et al. (2010) to the framework of temporal profiles of prospects. Our objective is to combine risk and time in a framework that allows for a first hand analysis of attitudes towards delayed risk.

A connection between the domain of risk and that of pure time preferences has been suggested before (Prelec and Loewenstein 1991, Dasgupta and Maskin 2005) and a specific role has been attributed to the treatement of probabilities (Quiggin and Horrowitz 1995, Halevy 2008). Recent contributions suggest that there may be subtle differences across these choice domains and that inverse-S weighting functions can play an important role for time (Wu 1999, Baucells and Heukamp 2012, Epper and Fehr-Duda 2015). For static risk settings the inverse-S shaped probability weighting functions have been supported empirically (Stott 2006, Wakker 2010) and several parametric forms have been suggested (Goldstein and Einhorn 1987, Lattimore et al. 1992, Tversky and Kahneman 1992, Prelec 1998, Diecidue et al. 2009). Our focus on the constant relative sensitivity (CRS) family of Abdellaoui et al. (2010) is also motivated by the empirical findings in Abdellaoui et al. (2011), paired with the arguments put forward by Gonzalez and Wu (1999). The latter suggested that the shape of inverse-S probability weighting function can be attributed to elevation and curvature, each measure reflecting independent psychological considerations that may affect the choice behavior of individuals. The former added a time dimensions and provided experimental evidence supporting a change in the degree of curvature when the resolution and outcome in a prospect are delayed.

Elevation is seen as a general index of confidence in making risky choices, whereas curvature indicates the ability of decision makers to discriminate between probabilities of outcomes (Gonzalez and Wu 1999). As argued in Abdellaoui et al. (2010), the CRS weighting functions use a specific parameter for each psychological consideration. Elevation can be thought to be a time independent component of individual's risk attitude towards sources of uncertainty (e.g., risk versus ambiguity as in Abdellaoui et al. 2011a). By contrast, curvature is likely to depend on temporal aspects. For instance, if an individual is relatively insensitive to small changes in probabilities for outcomes of intermediate rank within a prospect, as predicted by the inverse-S shaped weighting functions where more attention is given to extreme outcomes, it seems reasonable that such insensitivity is less pronounced when that prospect is resolved imminently and extreme outcomes receive most attention. Similarly, as a prospect is delayed and the outcomes are not affecting individuals immediately, it seems plausible that intermediate outcomes receive more attention in the evaluation process. As a consequence, an individual with more sensitivity resulting from delay may appear more tolerant to delayed risk relative to immediate risk. Indeed, recent studies confirm that many individuals seem to exhibit such behavior (Noussair and Wu 2006, Abdellaoui et al. 2011, Epper and Fehr-Duda 2015). Therefore, we propose an temporal version of the CRS weighting function of Abdellaoui et al. (2010) by keeping the elevation parameter constant over time but allowing the relative curvature parameter to vary.

Given the descriptive focus of our model, it is worth reviewing some of the experimental findings related to temporal risk attitudes which we seek to capture. Earlier studies in the psychology literature have focused on differences in discounting of delayed risky prospects relative to riskless outcomes using judgement tasks (Stevenson 1992) and, using rating tasks, differences in discounting of losses relative to gains in mixed prospects (Shelley 1994). The latter study finds more discounting for risky losses relative to gains, in contrast to the findings in Thaler (1991) for sure outcomes, a discrepancy that suggests more risk tolerance with delay.

The study of Keren and Roelofsma (1995) focuses on the immediacy effect, that is, the empirical observation of a strong preference for a sure outcome now relative to a somewhat larger delayed outcome which is reversed in preference when a common delay is added to both alternatives. In Experiment 1, they observe that the immediacy effect is significantly reduced when the outcomes are likely ($p = 0.9$) and the effect is next to non-existent if outcomes are risky ($p = 0.5$). In Experiment 2, Keren and Roelofsma replicate Kahneman and Tversky's (1979) common consequence choices in a between subject design and find that the common consequence effect persists when a common delay of one year is added. This suggests a role for inverse-S probability weighting functions as they can acommodate Allais (1953) common consequence paradox to EU-preferences.

Weber and Chapman (2005) replicate the study of Keren and Roelofsma (1995) and find that the immediacy effect is replicated for sure outcomes and that it persists when those same outcomes are risky ($p = 0.5$). Like Keren and Roelofsma, they find that the common consequence effect persists when a common delay is added. Weber and Chapman also consider the common ratio effect choices (the version in Kahneman and Tversky 1979), and find that adding a common delay does not have any measurable impact on the choice behavior in the common ratio tasks. They do, however, find that elicited certainty equivalents for the different prospects are affected by a common delay: certainty equivalents for the immediate prospects are in line with the common ratio effect while certainty equivalents for delayed prospects where too close to generate the common ratio effect. Baucells and Heukamp (2010) also study the effect of delay in common ratio choice tasks. They find that a preference for the sure outcome is significantly reduced by delay. These findings also suggest that subjects become more risk tolerant when prospects are delayed.

Noussair and Wu (2006) use choice lists in which the outcomes of two binary prospects are fixed and probabilities vary (Holt and Laury 2002). In prospect A the outcomes were close (\$10 versus \$8) while prospect B had outcomes more spread (\$19.50 versus \$0.50). Starting with probability 0.1 for the higher outcome (residual probability being given to the lower outcome) and repeatedly shifting probability mass in units of 0.1 from the lower to the higher outcome, an expected value maximizer would choose prospect A for all low probabilities of the higher outcome and change to B at the $50 : 50$ probability distribution (and choose the latter from there on). A more risk averse decision maker may swap from choosing A to choosing B at probabilities above 0.5 for the better outcome. Because the outcome-stimuli are of a small scale and are kept fixed within the choice lists, it is likely that attitudes towards probabilities, as captured in PT, may be the main driver for risk behavior.[1] When the resolution and payment of the prospects is delayed by three months, most subjects remain consistent in their choices or become more risk seeking: they start choosing prospects B at lower probabilities for the larger outcomes relative to the treatment when resolution and payment is immediate. Most subjects appear to be more risk tolerant when a received prospect's resolution is delayed.

Abdellaoui, et al. (2011) study choice behavior for prospects that are obtained and resoved at a common date. This enables them to separate effects attributed to discounting from the treatement of probabilities and attitudes towards outcomes over time much cleaner than in earlier studies. By adopting a general canonical model and elicitig temporal certainty equivalents for correspondingly timed prospects, they obtain data for utility and probability weighting functions for instant and delayed settings. They find that probability weights for $p = 1/3$ are not significantly impacted by time, that the low probability ($p = 1/6$) is less overweighted with delay, and that moderate and large probabilities ($p \in \{1/2, 2/3, 5/6\}$) are less underweighted when prospects are delayed. For utility they find no significant effect of time. The results suggest that subjects are more sensitive to changes in probabilities when the delayed prospects are evaluated, a behavior that implies more risk tolerance under PT.

Having mentioned some of the literature on empirical findings that we seek to acommodate in

[1] Assuming EU with a slightly concave utility for small scale outcomes (e.g., a power utility with power parameter of 0.88) is not sufficient to explain a preference for A when the best outcome has a probability larger than 0.5. Indeed, the consequence of a concave utility for small scale gains under EU is that an unreasonable degree of risk aversion for larger scale gains is implied, which has forcefully been criticized by Rabin (2000).

our model, it is worth listing a few experimental results that are related to resolution of uncertainty and other attitudes, but which fall outside the scope of our model. For instance, Ahlbrecht and Weber (1997) also consider preferences over risky prospects. Their study has fixed payment dates but distinct timing of the resolution of some of the uncertainty. They compare choices among gain prospects, where all resolution of uncertainty is early, gradual resolution where some uncertainty is resolved early and some late, or all resolution is late; similarly, they also implement choices among loss prospects. Ahlbrecht and Weber find that a large majority of their subjects have a consistent preference for the timing of resolution and that most subjects prefer early relative to late resolution of uncertainty when these are the only possibilities. They then compare these consistent subjects with their choices among the gradually resolved prospects and report various inconsistencies that cannot be accommodated by a transformation of risky utility as proposed in the Kreps and Proteus (1978) type models. A similar argument was presented in the study of Chew and Ho (1994). They observed risk attitudes in accordance with PT for instantly resolved uncertainty while, for temporal prospects, they report that risk attitude and attitude towards resolution of uncertainty do not seem correlated. As the timing of the payment is common in the prospects with gradually resolved uncertainty, discounting cannot explain the findings in these studies either. Indirectly these findings also give support for a model that allows for probability weighting to be time-dependent, as suggested in Wu (1999).

In our model we assume that the cardinal utility under risk is equal to the utility for intertemporal outcomes. Transferability of utility across risk and time has been adopted in Andersen et al. (2008) and Takeuchi (2011), who use the risky utility to measure discount rates (see also Frederick et al. 2002). Although in practice the transferability of utility across contexts is supposed, the assumption is not uncontroversial. Booij and van Praag (2009) show that the degree of risk aversion may be affected by time-preferences. Assuming EU-preferences for risk, Andreoni and Sprenger (2012) and Coble and Lusk (2010) find differences in the utilities for risk and for time. Accounting for non-EU preferences, Abdelaoui et al. (2013), who empirical measure risky and temporal utility for gains and losses, find that curvature and loss aversion are more pronounced for risk relative to time. Comparing risky utility revealed from choices with utility under certainty derived from strength of preference judgements, Abdellaoui et al (2007) find little difference when accounting for potential biases attributed to the treatment of probabilities under risk.

Abdellaoui et al. (2013) use a specific revealed preference method designed to circumvent biases due to nonlinear treatment of probabilities or non-constant discounting. Revealed preference methods have tradition in neoclassical consumer theory and have also been adopted in laboratory studies to elicit risky utility from choices framed in an asset demand setting (Choi et al. 2007, Polisson et al. 2017). Sometimes those techniques are combined with incentive schemes that invoke a temporal element (Andreoni and Sprenger 2012). In contrast to the laboratory, real data on asset demands is not collected instantly. Instead, it is obtained at various points in time, and one can plausibly assume that changes in asset prices reflect, at least in part, changes in the underlying subjective probabilities held over the assets' payoffs (unless markets are in equilibrium). When extracting information about the risky utility from such real data, one implicitly assumes a common utility for risk and time. As Kübler et al. (2014) show, when state-probabilities are allowed to vary, such an assumption is not innocent as additional conditions for revealed preferences must be satisfied to ensure that asset demands correspond to choices resulting from the maximization of EU-preferences. Although Kübler et al. do not model time explicitly, their argument applies when the laboratory techniques are transferred to be used for real data analysis; see also Kübler et al. (2017).

As our theory is intended for the study of today's preferences over risky profiles and to understand if delay has an effect on attitudes towards probabilities, we feel the assumption of a common utility (i.e., today's risky utility) being used in the evaluation of profiles of risky prospects is in order. When studying preferences at different time points one would need to account for tastes that may change over time, which could well be captured by utility. A corresponding framework would be much richer in the degrees of freedom needed to explain inconsistent behavior. We model instant decision over (streams of) objects that are received and resolved at the same time. For such choices, in the afore-mentioned study of Abdellaoui et al. (2011), more risk tolerance for delayed risky prospects was observed but no evidence for a significant impact of time on utility was documented. Indeed, Abdellaoui et al. conclude that probability weighting is accountable for all of the effect of delay and suggested that the increase in risk tolerance relative to immediately resolved risk is attributable to a more optimistic perception of probabilities.

For a standard probability weighting function, optimism is reflected as convexity and pessimism as concavity (Wakker 1994). PT-weighting functions are special in that they combine optimism for small probabilities of good outcomes and pessimism for medium to large probabilities of lower ranked outcomes (Tversky and Kahneman 1992, Wakker 2010). Of these, the parametric weighting function of Abdellaoui et al. (2010) cleanly separates relative optimism from a generic propensity towards risk taking. Given these features and the empirical findings listed above, we think that, as a first approximation, this simple parametric form is suitable to theoretically analyse the potential implications of time delay on the treatment of probabilities, in particular the ability to discriminate between probabilities and the implied probabilistic risk behavior.

Next we present our theoretical framework and the assumptions for temporal risk preferences. A foundations for time-dependent CRS probability weighting is provided in Section 3. Subsequently we provide some comparative analyses, some simple applications to bargaining and conclude. Proofs are presented in the Appendix.

2 Theoretical Framework

Initially, we present notation for risky objects in a timeless setting and the general preference model used to evaluate these. Then we discuss the specific probability weighting functions adopted in our temporal model. Subsequently, the more general setting is presented in which profiles of lotteries obtained and resolved at specified dates are defined. Following that, we introduce the temporal model with time-dependent constant relative sensitivity weighting functions.

2.1 Risky Outcomes

Let $\mathbb{R}_+$ denote the set of non-negative deterministic *outcomes*. General outcomes are denoted $x, y, z, \ldots$, while in specific cases we use $a, b, c, d, \ldots$ to express properties of preferences. A *prospect* is a finite probability distribution over outcomes and is denoted as $\tilde{x} = (p_1 : x_1, \ldots, p_n : x_n)$ meaning that outcome $x_j \in \mathbb{R}_+$ is obtained with probability p_j, for $j = 1, \ldots, n$. As usual, $p_j \geq 0$ for all $j = 1, \ldots, n$ and $\sum_{i=1}^{n} p_i = 1$ is assumed. Let $\mathcal{L}$ denote the set of all prospects.

For convenience of notation we always write prospects with the outcomes ordered from best to worst, i.e., for $\tilde{x} = (p_1 : x_1, \ldots, p_n : x_n)$ we have $x_1 \geq \cdots \geq x_n$. Further, we identify sure outcomes with the corresponding degenerate prospect; also, for $\tilde{x} = (p_1 : x_1, \ldots, p_n : x_n) \in \mathcal{L}$, the set of prospects with the probabilities $\{p_1, \ldots, p_n\}$ fixed is denoted by $\mathcal{L}_{\{p_1,\ldots,p_n\}}$. By $a_j\tilde{x}$ we denote the lottery $\tilde{x} = (p_1 : x_1, \ldots, p_n : x_n)$ with outcome x_j replaced by $a \in \mathbb{R}_+$ if $p_j > 0$ for $j \in \{1, \ldots, n\}$. Given the ordering of outcomes within prospects, the implicit constraint $x_{j-1} \geq a \geq x_{j+1}$ applies. This ensures that both $a_j\tilde{x}$ and $\tilde{x}$ are lotteries in $\mathcal{L}_{\{p_1,\ldots,p_n\}}$.

2.2 Prospect Theory for Risky Outcomes

In our models below, we adopt *prospect theory*, PT for short (Tversky and Kahneman 1992) with an inverse-S probability weighting function. As we do not treat outcomes as gains or losses relative to a fixed reference point, our model boils down to rank-dependent utility (RDU; Quiggin 1982, Segal 1987, Wakker 1994), though much of the literature refers to this as PT because of the specific form of the probability weighting function; we follow this convention. Under PT, the value of a prospect is $\tilde{x} = (p_1 : x_1, \ldots, p_n : x_n) \in \mathcal{L}$ is given by

$$PT(\tilde{x}) = \sum_{j=1}^{n} [w(p_1 + \cdots + p_j) - w(p_1 + \cdots + p_{j-1})]u(x_j), \tag{1}$$

where ($p_0 := 0$ and) $p_j^* := p_1 + \cdots + p_j$ are cumulated probabilities for $j = 1, \ldots, n$ and w is a *probability weighting function* on the unit interval, i.e., $w : [0, 1] \to [0, 1]$ is strictly increasing and continuous and it satisfies $w(0) = 0$ and $w(1) = 1$. Further, u is a *utility* function, i.e., $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing and continuous. In Eq. (1) the probability weighting function is unique and the utility function is *cardinal*, i.e., unique up to multiplication by a positive constant and addition of a real number. If $w(p) = p$ for all $p \in [0, 1]$, then PT reduces to *expected utility* (EU).

2.3 Constant Relative Sensitivity Weighting

In analogy to how power utility functions are related to constant absolute risk aversion in EU, power probability weighting functions are related to modeling risk attitudes related to probabilities. This was exploited in Abdellaoui et al. (2010) to obtain the *constant relative sensitivity* (CRS) weighting functions, which are defined as follows:

$$w(p) = \begin{cases} \eta^{1-\sigma} p^{\sigma}, & \text{if } p \in [0, \eta], \\ 1 - (1-\eta)^{1-\sigma}(1-p)^{\sigma}, & \text{if } p \in (\eta, 1], \end{cases} \tag{2}$$

where $\eta \in [0, 1]$ is the parameter for *elevation* and $\sigma > 0$ is the *(in)sensitivity* parameter. To ensure the empirically documented inverse-S shape for the CRS weighting function, we require $0 < \eta < 1$ and $\sigma < 1$. If $\eta = 1$ then the CRS-weighting function in Eq. (2) is a power function that is concave over the entire probability interval, hence exhibiting *optimism* (Wakker 1994); for $\eta = 0$ we have *pessimism* as the CRS-weighting function is convex. Our restrictions on η ensure that optimism is exhibited for small probabilities ($p < \eta$) of good outcomes and pessimism for larger probabilities of less good outcomes ($p > \eta$).

As shown in Abdellaoui et al. (2010), the index of curvature of the CRS weighting function can be measured using the analogous of the Arrow-Pratt coefficient of relative risk aversion (Arrow 1971, Pratt 1964). For the CRS weighting function this index is constant and equals $1 - \sigma$, hence it is positive and bounded when the shape of the probability weighting function is inverse-S. For intermediate probabilities away from 0 and 1 and close to η the CRS function can be approximated by a linear weighting function that has slope σ and fixed point $p = \eta$, and which is discontinuous at 0 and at 1. Such probability weighting functions are referred to as NEO-additive as they induce preferences close to EU at non-extreme outcomes but give extra weight to best and worst outcomes (Chateauneuf, Eichberger and Grant 2007, Webb and Zank 2011). The constant slope σ or the curvature index $1-\sigma$ can therefore be used for comparative interpersonal analyses and, in particular, for intrapersonal comparative statics concerning the changes in sensitivity resulting from a temporal delay of prospects.[2] To this aim we expand our static framework and consider profiles of risky outcomes.

[2]For empirically founded intrapersonal comparisons of insensitivity to choice based probabilities derived using different source of uncertainty, Abdellaoui et al. (2011a) have adopted NEO-additive probability weighting functions.

2.4 Risky Profiles

In this paper we consider preferences over profiles of risky outcomes. We assume discrete time periods $t = 0, \ldots, T$ for $T \geq 1$. The objects of choice are risky *profiles*, i.e., profiles of prospects that are obtained and resolved at the indicated time period. We use bold-faced letters to denote profiles, that is, $\mathbf{x} = (\tilde{x}_0, \tilde{x}_1, \ldots, \tilde{x}_T)$ is a profile where prospect $\tilde{x}_t = (p_{t,1} : x_{t,1}, \ldots, p_{t,n_t} : x_{t,n_t})$ is obtained and resolved at time t for all $t \in \{0, \ldots, T\}$. The set of all profiles is $\mathcal{P} = \Pi_{t=0}^{T} \mathcal{L}$. Sometimes we write $(\tilde{x}_t)\mathbf{x}$ or simply $\tilde{x}_t\mathbf{x}$ to highlight the prospect obtained at time t.

A preference relation is a binary relation $\succcurlyeq$ defined over $\mathcal{P}$, with $\succ$ denoting strict preference, and $\sim$ denoting indifference; reversed symbols denote corresponding preference, as usual. A real-valued function V *represents* $\succcurlyeq$ on $\mathcal{P}$ if $\mathbf{x} \succcurlyeq \mathbf{y} \Leftrightarrow V(\mathbf{x}) \geq V(\mathbf{y})$ for all profiles $\mathbf{x}, \mathbf{y} \in \mathcal{P}$. Profiles of prospects are evaluated by discounted prospect theory. We present this general model within a formal definition.

Definition 1. General *Discounted Prospect Theory* (DPT) holds if the preference relation $\succcurlyeq$ on $\mathcal{P}$ is represented by

$$DPT(\mathbf{x}) = \sum_{t=0}^{T} D(t) PT_t(\tilde{x}_t), \tag{3}$$

where the *discount function*, $D(\cdot)$, is positive valued with $D(0) = 1$, and PT_t stands for time-dependent prospect theory, which in general has a time-dependent probability weighting function, w_t, and a corresponding utility function, u_t, for $t = 1, \ldots, T$. The specific normalization of the discount function renders it unique as are the weighting functions, while the utiliy functions are jointly cardinal (i.e., they can be replaced by $v_t = Au_t + B_t$ for a positive A and real valued B_t, $t = 0, \ldots, T$).

DPT differs from the classical constant discounted expected utility (DEU) model as it allows for a general discounting function. In applications usually one assumes genuine discounting, i.e., $D(t) > D(s)$ for all $s, t \in \{0, \ldots, T\}$ with $t < s$, or adopt specific parametric forms like exponential discounting (Samuelson 1937, Koopmans 1960), quasi-hyperbolic discounting (Phepls and Polak 1968, Laibson 1997), or more general parametric families as discussed in Bleichrodt et al. (2009). A further aspect where DPT departs from classical DEU is the evaluation of prospects, where in DPT the most successful descriptive model for static choice, PT (Wakker 2010, Barberis 2013), is used. That DPT is quite general also follows from the fact that, without invoking further constraints on behavior, the weighting functions and utility functions at different time points can be unrelated.

The preference relation $\succcurlyeq$ on $\mathcal{P}$ induces a corresponding relation, $\succcurlyeq_t$, over prospects obtained at time $t = 0, \ldots, T$, which in turn induces a preference relation over timed outcomes; we use the same symbol, $\succcurlyeq_t$, for the latter. As the preference considered in Definition 1 are additively separable over time periods, the restrictions of the preference relation to individual time periods are well-defined. While DPT allows for general probability weighting functions to depend on time, the specific version we adopt here has further restrictions as stated in the next assumption.

Assumption 1. *Throughout we assume that DPT in Eq. (3) has CRS probability weighting functions in each time period. That is, we assume that, for each induced preference relation* $\succcurlyeq_t$ *over prospects obtained at time* $t = 0, \ldots, T$*, the preference conditions of Abdellaoui et al. (2010) are satisfied.*

The preceding assumption allows for each induced preference relation $\succcurlyeq_t, t = 0, \ldots, T$, to be represented by PT with a CRS-probability weighting function. The assumption does not impose further relationships between the CRS parameters across time periods nor for the utility functions which, in general, can depend on the delay time $t = 0, \ldots, T$. We are, however, interested in some form of consistency across time periods. To obtain such a connection we require specific preference conditions, which we present in the next section.

3 Preference Properties

This section presents additional preference conditions to further restrict the general class of DPT representations. From Definition 1 one can infer that $\succcurlyeq$ on $\mathcal{P}$ is a complete and transitive preference (which is necessary for the existence of a representing function) that satisfies continuity and monotonicity in outcomes (as the discount function is positive valued, the weighting functions are strictly increasing and the utility functions are strictly increasing and continuous in each period). Further, according with our Assumption 1, the preference also satisfies continuity in probabilities as CRS weighting functions are assumed to be continuous on the probability interval. The next subsection focuses on properties for utility and the elevation parameters. Subsequently, we consider properties that relate to the curvature parameter of the CRS probability weighting function.

3.1 Time-invariant Utility and Elevation

The next preference condition is a consistency requirement for utility across time periods. It demands that riskless outcome-tradeoffs measured at different time periods are invariant to delay. Similar conditions have been used in the derivation of cardinal utility in static decision models for risk and ambiguity (Köbberling and Wakker 2003). We say that the preference relation $\succcurlyeq$ on $\mathcal{P}$ satisfies *time-invariance for outcome tradeoffs* if for all time periods $t, s \in \{0, \ldots, T\}$, $s < t$, all outcomes $a, b, c, d \in \mathbb{R}_+$ and all profiles $\mathbf{x}, \mathbf{y} \in \mathcal{P}$ any three of the following indifferences imply the fourth:

$$\begin{aligned} a_s\mathbf{x} \sim b_s\mathbf{y}, &\quad c_s\mathbf{x} \sim d_s\mathbf{y}, \\ a_t\mathbf{x} \sim b_t\mathbf{y}, &\quad c_t\mathbf{x} \sim d_t\mathbf{y}. \end{aligned}$$

Substitution of DPT into the preceding four indifferences, taking differences of the first pair of resulting equations and similarly of the second pair, and cancelling of common terms, one obtains the following utility differences

$$u_s(a) - u_s(b) = u_s(c) - u_s(d)$$
$$u_t(a) - u_t(b) = u_t(c) - u_t(d),$$

which are supposed to hold for all time periods $t, s \in \{0, \ldots, T\}$, $s < t$, and all outcomes $a, b, c, d \in \mathbb{R}_+$. That is, whenever the first equation holds also the second must hold, and it means that the continuous and strictly increasing utility functions u_s and u_t are proportional and can be taken equal. We obtain the following result.

Proposition 1. *Assume that the preference relation $\succcurlyeq$ on $\mathcal{P}$ is represented by DPT as in Definition 1. Then, $\succcurlyeq$ on $\mathcal{P}$ satisfies time-invariance for outcome tradeoffs if and only if $u := u_0 = u_t$ for all time periods $t \in \{1, \ldots, T\}$.* □

The proof of this proposition follows directly from the results of Köbberling and Wakker (2003); their results are tailored for ambiguity but their arguments apply similarly for our framework with risky profiles. Köbberling and Wakker show (see their Corollaries 29 and 10) that the analogous condition of time-invariance for outome tradeoffs is a powerful property which, for a representation that is continuous and strictly monotonic in outcomes, implies additive separability across time periods; subsequently, the proportionality results for utility are derived. The advantage of using outcome tradeoffs as a preference condition is that the tradeoff tool leans itself to the measurement of utility and this allows for non-parametric tests that can be used to empirically verify the time independence of utility under DPT (Wakker and Deneffe 1996, Abdellaoui 2000, Abdellaoui et al. 2010).

Our next property requires further consistency of preference across time periods as it ensures that the elevation parameter of the CRS probability weighting functions is also time-invariant. As argued in the introduction, the elevation parameter is regarded as an index that measures a general propensity of a decision maker to take risks. Like risk attitudes captured in the time invariant utility function, it is conceivable that the propensity to take risks is not affected by delay (although this remains an empirical question). Accordingly, by adopting the terminology of Gonzalez and Wu (1999), we say that the preference relation $\succcurlyeq$ on $\mathcal{P}$ satisfies *time-invariant propensity to gamble* if for all time periods $t, s \in \{0, \ldots, T\}$, $s < t$, all outcomes $a, b, x, y \in \mathbb{R}_+$ and all profiles $\mathbf{x} \in \mathcal{P}$ the following holds:

$$(\eta_s : a, 1 - \eta_s : x)_s\mathbf{x} \sim (\eta_s : b, 1 - \eta_s : y)_s\mathbf{x} \Rightarrow (\eta_t : a, 1 - \eta_t : x)_t\mathbf{x} \sim (\eta_t : b, 1 - \eta_t : y)_t\mathbf{x}.$$

Time-invariant propensity to gamble says that, for binary prospects where the probability of the better outcome is not distorted, hence, locally neither optimism nor pessimism can be inferred, only the tradeoffs among outcomes govern choice behavior. There is some empirical support for probabilities that are not distorted, in particular many studies report that probabilities close to $1/4 - 1/3$ are less subjected to distortion (Tversky and Kahneman 1992, Wu and Gonzalez 1996, Abdellaoui 2000, Bleichrodt and Pinto 2000, Abdellaoui et al. 2005, Etchard-Vincent 2004; see also the discussion in

Wakker 2010, Chapter 7). Abdellaoui et al. (2010) find no statistically significant difference between the elevation parameters of the CRS-weighting functions for gains and losses, and Abdellaoui et al. (2011, Tables 5&6) find no statistical significant differences between the distorted probability of $p = 1/3$ for delayed prospects. Both findings suggest that behavior captured through the elevation index is relatively stable across choice contexts (gains/losses, instant/delayed), and these findings are compatible with our preference requirement and with the property of the CRS-probability weighting functions that have a fixed point at η_t, i.e., $w_t(\eta_t) = \eta_t$ for all $t = 0, \ldots, T$.[3]

By additionally invoking the time-invariance property for the propensity to gamble in Proposition 1 we obtain the DPT model with time-invariant utility and time-invariant elevation index.

Proposition 2. *Assume that the preference relation* $\succcurlyeq$ *on* $\mathcal{P}$ *satisfies Assumption 1 and that time-invariance for outcome tradeoffs holds. Then,* $\succcurlyeq$ *on* $\mathcal{P}$ *satisfies time-invariant propensity to gamble if and only if* $\eta := \eta_0 = \eta_t$ *for all time periods* $t \in \{1, \ldots, T\}$. □

Proposition 2 provides a characterization of a special case of DPT with CRS probability weighting and constant elevation parameter across time; we label it DPT^η. In contrast to the elevation parameter and the utility of outcomes, we do not restrict the curvature parameter of the probability weighting functions in the DPT^η model. This allows for intrapersonal comparisons of the changes to curvature across time periods. We present this analysis in the next subsection.

3.2 Insensitivity and Delay

This subsection relates changes in the curvature of the probability function when risks are delayed to risk tolerance. We proceed by recalling some examples of choices among prospects used in the experimental literature, some of which was briefly touched upon in the Introduction.

The first example is due to Baucells and Heukamp (2010). They tested the effect of delay on the common ratio paradox to EU of Allais (1953). In one experiment (Baucells and Heukamp, Table 1) the choices were among two pairs of prospects that were obtained at time $t = 0$ (now), $t = 1$ (1 month), $t = 2$ (3 months), as follows (payments in EURO):

$$A = (1 : 9)_t \qquad \text{versus} \qquad B = (0.8 : 12, 0.2 : 0)_t$$

and

$$A' = (0.1 : 9, 0.9 : 0)_t \qquad \text{versus} \qquad B' = (0.08 : 12, 0.92 : 0)_t.$$

For $t = 0$ they find that 58% favor A in the first choice while 78% prefer B' in the second choice. For $t = 1$ the corresponding proportion of choices are 55% and 74%, and when $t = 2$ the respective proportions are 43% and 79%. The results suggest that delay induces many individuals to change

[3] Wakker (2010, pp. 205–206) reports that $w(1/3)$ being approximately 1/3 is the most common finding for probability weighting w under PT.

their preferece from $A \succ B$ at time $t = 0$ to $B \succ A$ at time $t = 2$. This shift in preference seem to happen gradually as a delay of 1 month does lead to just minor changes in the proportions, such that an immediacy effect as explanation of the common ratio effect can be excluded here. Moreover, as the proportions of choices for B' in the second pair of prospects are relatively stable, it seems as if the subjects in this experiment are better at discriminating between probabilities 0.8 and 1.0 when the choices are delayed, even though the common ratio effect persists, albeit less pronounced.

The implications of delay for the common consequence effect of Allais (1953) was also tested in Weber and Chapman (2005). The choices in their experiment were —using the Kahneman and Tversky (1979) version— among two pairs of prospects that were obtained at time $t = 0$ (now), $t = 1$ (1 year), $t = 2$ (25 years), as follows (payments in US$):

$$C = (1 : 2700)_t \quad \text{versus} \quad D = (0.33 : 3000, 0.66 : 2700, 0.01 : 0)_t$$
$$\text{and}$$
$$C' = (0.66 : 2700, 0.34 : 0)_t \quad \text{versus} \quad D' = (0.33 : 3000, 0.67 : 0)_t.$$

They find that the common consequence effect persist when prospects are delayed although it becomes somewhat weaker when $t = 1$. That is, for $t = 0$ prospect C is chosen by 85% of the subjects while 58% favor D'; for $t = 1$ and $t = 2$ the corresponding percentages are 76% and 60% and, respectively, 82% and 57%. While a time delay of 25 years adds a lot of uncertainty and may lead subjects to adhere to current preferences, the delay of 1 year suggests that individuals have a slight tendency to account for small probability differences (1.0 and 0.99) of gaining a large sum of money, which indirectly means that they better discriminate between those large probabilities.

The preceding two studies have indicated that changes in preference as a result of delay are most likely to be observed when the choice is between a sure or very likely outcome and a non-degenerate prospect. Accordingly, our third summary example looks at the more recent study of Abdellaoui et al. (2011) who emply such choices. More specifically, they elicited certainty equivalents for binary prospects while varying outcomes and probabilities, and they considered settings with no delay ($t = 0$), 6 months delay ($t = 1$) and a delay of one year ($t = 2$).[4] Their initial finding is that certainty equivalents tend to increase with delay, indicating more risk tolerance as suggested by Noussair and Wu (2006). Such risk tolerance can be the result of better discrimination between probabilities when prospects are delayed.

A further finding of Abdellaoui et al. (2011), based on the assumption that utility is a power function as in Tversky and Kahneman (1992), was that statistically there is no significant difference between the elicited power parameters for utility. This means that the observed risk tolerance can

[4] Abdellaoui et al. (2011) also used a setting with an unspecified or "ambiguous" date of delay between 6 and 12 months. Such ambiguity adds an additional consideration into the decision making process, not captured by DPT, and seems to lead to more risk aversion than observed for the clearly specified dates of 6 and 12 months.

mainly be attributed to the treatement of probabilities. Indeed, Abdellaoui et al. find that the weights for probabilities $p \geq 1/2$ increase with delay but remain relatively unchanged for $p \in \{1/6, 1/3\}$. Both, the findings for utility and for probability weighting are in line with the assumptions underlying the DPT model suggested in Proposition 2. Therefore, in order to explain the phenomenon of more risk tolerance as a result of delay, we have just one more degree of freedom left in our model, namely the curvature parameter σ of our CRS probability weighting functions.

Next we proceed with intrapersonal analysis on the empirically observed phenomenon of more sensitivity due to delay. We adopt the probability midweight tool of van de Kuilen and Wakker (2011). This tool has recently been adopted by Werner and Zank (2017) to provide preference foundations for PT in a static framework without prior knowledge of the location for the reference point. Werner and Zank indicate that the probability midweight method is suitable for empirically detecting reference point effects beyond it being a non-parametric way of eliciting probability weighting effects and its appeal for comparative analyses.

The midweight method of van de Kuilen and Wakker (2011) requires the prior elicitation of a utility midpoint before proceeding with the elicitation of probabilities. Given the continuity assumptions for utility and probability weighting functions under DPT as in Proposition 2, such midpoints for utility and weighting functions are always feasible and well-defined. Hence, we can fix outcomes $x > y > 0$ such that $u(x) - u(y) = u(y) - u(0)$ and find, for some $t \in \{0, \ldots, T-1\}$, the probability p_t such that

$$(\eta : x, 1-\eta : y)_t\mathbf{z} \sim (p_t : x, 1-p_t : 0)_t\mathbf{z}.$$

Then, adding a delay of one time period leads to reduced insensitivity (equivalently, increased sensitivity) if

$$(\eta : x, 1-\eta : y)_{t+1}\mathbf{z} \sim (p_{t+1} : x, 1-p_{t+1} : 0)_{t+1}\mathbf{z}$$

implies that $p_t > p_{t+1}$. If this implication holds for all $t \in \{0, \ldots, T-1\}$ we say that $\succcurlyeq$ on $\mathcal{P}$ satisfies *increasing sensitivity with delay*.

To further clarify the implications of delay on the sensitivity paramenter of the CRS probability weighting functions, we proceed with some derivations resulting from substitution of DPT^{η} into the first of the preceding indifferences. After cancelling the common terms related to $\mathbf{z}$ outside period t, we obtain

$$w_t(\eta)u(x) + [1 - w_t(\eta)]u(y) = w_t(p_t)u(x) + [1 - w_t(p_t)]u(0).$$

Next, exploiting that η is a fix-point of w_t and that y is a utility midpoint between 0 and x, we obtain

$$w_t(p_t) = \frac{1+\eta}{2}, \tag{4}$$

such that p_t is a *midweight* between η and 1 for w_t (i.e., a probability midpoint on the w_t-scale).

Similarly, using DPT^{η} in the second indifference, we obtain that p_{t+1} is a midweight between η and 1 for w_{t+1}. We conclude that increasing sensitivity to delay implies that $p_t > p_{t+1}$ and $w_t(p_t) = w_{t+1}(p_{t+1})$. The corresponding implication for the CRS weighting functions of adjacent time periods can be seen in Figure 1, where the horizontal axis depicts cumulated probabilities which are weighted by w_t, respectively, w_{t+1} to values on the vertical axis:

Figure 1: Effect of Increasing Sensitivity to Delay

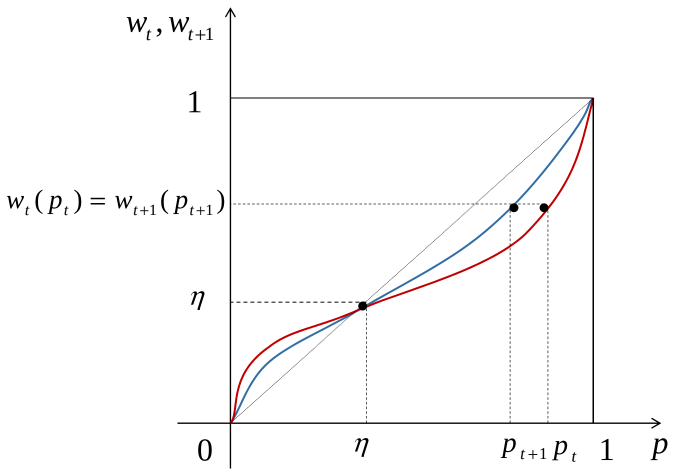

Figure 1 indicates, that the requirement for the midweight p_{t+1} to be smaller than p_t implies that w_{t+1} is closer to the 45-degree line (where $w_s(p) = p$ as in EU) than w_t is. This means that w_{t+1} is steeper than w_t, hence, is more sensitive to changes in probabilities. This can also be inferred from substitution of the CRS-probability weighting function in Eq. (4). As $\eta < p_{t+1} < p_t < 1$, one obtains

$$1 - (1-\eta)(1/2)^{1/\sigma_{t+1}} = p_{t+1} < p_t = 1 - (1-\eta)(1/2)^{1/\sigma_t}$$

from which $\sigma_t < \sigma_{t+1}$ follows as a result of reduced insensitivity with delay.

Next, we discuss the empirically observed phenomenon of more risk tolerance with delay. Recall the example of choices in Baucells and Heukamp (2010) discussed above. It appears as if at time 0 individuals have a preference for $A \succ_0 B$ and at a later date $t > 0$ this is reversed to $B \succsim_t A$. Such behavior suggests that the the preference for the sure A today is changing as a result of delay to a preference for the risky B. Thus, an increased willingness to take risk is observed when prospects are obtained later. Formally, we say that $\succcurlyeq$ on $\mathcal{P}$ exhibits *increased risk tolerance with delay* if for all outcomes $x > y > 0$, all profiles $\mathbf{z} \in \mathcal{P}$, all probabilities $\eta < p < 1$ and all time periods $t \in \{0, \ldots, T-1\}$ we have

$$(p : x, 1-p : 0)_t\mathbf{z} \sim (1 : y)_t\mathbf{z} \Rightarrow (p : x, 1-p : 0)_{t+1}\mathbf{z} \succ (1 : y)_{t+1}\mathbf{z}.$$

Increased risk tolerance is formulated for probabilities where the CRS weighting functions display pessimism, i.e., they are convex shaped (Chew, Karni and Safra 1987, Chateauneuf and Cohen 1994, Wakker 1994, 2010, Baucells and Heukamp 2006, Ryan 2006, Schmidt and Zank 2008). For positive probabilities smaller than η the CRS weighting functions exhibit optimism, i.e., risk proneness relative to EU-preferences as revealed through concavity of the probability weighting function. It seems less plausible to demand or detect more risk tolerance over a domain of prospects where the typical behavior would be more risk seeking relative to EU. For this reason, increased risk tolerance due to delay is defined for choices among binary prospects where the typical finding is risk aversion. Substitution of DPT^{η} into the preceding preferences implies that there must be less convexity of the CRS weighting function as a result of delay. This means that $\sigma_t < \sigma_{t+1}$ for all $t \in \{0, \ldots, T-1\}$. We summarize the analysis of this section in the following theorem.

Theorem 1. *Assume that $\succcurlyeq$ on $\mathcal{P}$ is represented by DPT^{η} as in Proposition 2. Then the following statements are equivalent:*

(i) *The preference $\succcurlyeq$ on $\mathcal{P}$ satisfies increased sensitivity with delay;*

(ii) *The preference $\succcurlyeq$ on $\mathcal{P}$ satisfies increasing risk tolerance with delay;*

(iii) *For all $t \in \{0, \ldots, T-1\}$ we have $0 < \sigma_t < \sigma_{t+1} < 1$.*

□

The data in Abdellaoui et al. (2011) indicate that indeed the subjects in their study exhibit more risk tolerance with delay of prospects. They inferred this through the elicitation of certainty equivalents for binary prospects. Theorem 1 indicates alternative means of detecting risk tolerance through the identification of midweights, the non-parametric tool proposed by van de Kuilen and Wakker (2011). Empirically, it is unclear if the monotonic relationship for the sensitivity parameter of the CRS weighting function is valid for all conceivable time periods. As the example in Weber and Chapman (2005) suggests, a delay of 25 years seems to lead to more insensitivity for moderate probabilities. Such a distant time period may indeed induce subjects not to worry about small changes in probabilities as the outcome of those prospects will affect them only in the very distant future, if at all (for instance, survival considerations may play a role, which are not part of our DPT model).

4 An Application to Bargaining

In this section, we provide an application of our model to bargaining. In doing so, we demonstrate how increasing tolerance to risk due to delay introduces interesting dynamic issues that do not arise under discounted expected utility. A central result of noncooperative bargaining theory (Rubinstein 1982) is that agreements will be reached immediately. The reason for this prediction is that delay is costly to all parties involved in the bargaining process. Delays may nonetheless occur and a literature has emerged that attempts to explain how delays in agreement can result. Most explanations for bargaining delays have appealed to asymmetric information (Rubinstein 1985, Gul and Sonnenschein 1988, Abreu and Gul 2000). Other explanations have considered alternative assumptions regarding the opportunity costs of delay (Fernandez and Glazer 1991), the possibility of credible threats to reduce the available surplus (Avery and Zemsky 1994), and the way disagreement outcomes are decided (Busch and Wen 1995).

In the standard cooperative setting, assuming DEU, delay has a similar effect of inducing immediate agreement: the timing of the bargaining and the timing of the outcome are immaterial for reaching a solution. This may explain why in the cooperative setting invoking a richer setup with time as an additional dimension was required. For the more general preferences discussed in this paper, new possibilities arise, hence we need to expand the standard cooperative setting. Suppose that two individuals, labeled A and B, are bargaining over the chance to receive an indivisible rent R obtained at time $\tau \in \{0, \dots, T\}$. Disagreement results in outcome d for each individual. The actual bargaining takes place at time $t_b \in \{0, \dots, \tau\}$, a period that we assume exogenously given. The fact that individuals are bargaining at time t_b over an outcome received at time τ introduces a possible bargaining outcome delay, as measured by $\tau - t_b$. We ask the question if this delay is advantageous and compare the result with the classical DEU case.

We next assume that both individuals maximise DPT with constant relative sensitivity (i.e., DPT^η for individuals A, B with respective discount functions D_A, D_B, utilities u_A, u_B and weighting functions $w_{A,t}, w_{B,t}$ with corresponding parameters η_A, η_B and $\sigma_{A,t}, \sigma_{B,t}$). After normalising utilities so that $D_A(\tau)u_A(R) = D_B(\tau)u_B(R) = 1$ and $D_A(\tau)u_A(d) = D_B(\tau)u_B(d) = 0$. The bargaining set in utility space with bargaining outcome time τ and bargaining time t_b is:

$$\mathcal{B}_\tau^{t_b} = \{(w_{A,\tau-t_b}(p_A), w_{B,\tau-t_b}(p_B)) : p_A, p_B \geq 0, p_A + p_B \leq 1\},$$

with the interpretation that either individual A obtains R with probability p_A, or individual B obtains R with probability p_B at time τ (else 0 utility is obtained).

A few potential solutions to the one-shot bargaining problem could be considered, and we follow Köbberling and Peters (2003) who argued that for PT-preferences —like the one considered here— the Kalai-Smorodinsky (KS) solution is appropriate. Existence is guaranteed, as the Pareto frontier of the bargaining set in utility space is strictly decreasing (Conley and Wilkie 1991). One can then show that, in utility space, the KS-solution is given by:

$$KS(\mathcal{B}_{\tau}^{t_b}, (0,0)) = \{(w,w) : w_{A,\tau-t_b}(p) = w_{B,\tau-t_b}(1-p) = w, p \in [0,1]\},$$

which is unique and well-defined.

With bargaining time being t_b and bargaining outcome time being τ various scenarios emerge allowing for comparative analyzes. For expositional reasons and to maintain simplicity, we consider the case of two periods, 0 and 1, only, and we compare the results of DPT$^{\eta}$-preferences with those implied by DEU.

Claim 1. Let discounted expected utility hold. For a given bargaining time, bargaining outcome delays are never beneficial. For a given bargaining outcome time, bargaining delays have no effect. □

The following table illustrates the statements in Claim 1.

Table 1: Bargaining Outcomes under DEU.

Bargaining/Outcome time	Utilities at time $t = 0$
$t_b = 0, \tau = 0$	$(1/2, 1/2)$
$t_b = 0, \tau = 1$	$(D_A(1)/2, D_B(1)/2)$
$t_b = 1, \tau = 1$	$(D_A(1)/2, D_B(1)/2)$

In the table above, the change from $t_b = 0, \tau = 0$ to $t_b = 0, \tau = 1$ corresponds to a bargaining outcome delay. Because for both individuals the discount function is strictly decreasing under DEU, it follows that $D_i(1) < D_i(0) = 1$, which implies $D_i(1)/2 < 1/2$, for $i = A, B$. Thus, if individuals have a choice, they would be best placed to bargain in period 0 and reach immediate agrement with outcomes paid out at the same time. To see that, for a given outcome time, bargaining time delays have no effect, we compare the cases in the last two columns of Table 1. We observe that for both scenarios ($t_b = 0, \tau = 1$ and $t_b = 1, \tau = 1$) the utilities are unchanged at $(D_A(1)/2, D_B(1)/2)$. For DPT$^{\eta}$-preferences the results are different, as we state in our next claim.

Claim 2. Let DPT hold with increasing risk tolerance. Then, bargaining outcome delays can be beneficial. □

The following Table illustrates the possible outcomes under the assumptions of Claim 2.

Table 2: Bargaining Outcomes under DEU.

Bargaining/Outcome time	Utilities at time $t = 0$
$t_b = 0, \tau = 0$	(w, w) with $w_{A,0}(p) = w_{B,0}(1-p) = w$
$t_b = 0, \tau = 1$	$(D_A(1)w, D_B(1)w)$ with $w_{A,1}(p) = w_{B,1}(1-p) = w$
$t_b = 1, \tau = 1$	$(D_A(1)w, D_B(1)w)$ with $w_{A,0}(p) = w_{B,0}(1-p) = w$

To give a concrete example, suppose that individual A maximises DEU (i.e., DPT$^\eta$ with $\sigma_A = 1$) and individual B is purely pessimistic ($\eta_B = 0$), and let $\sigma_{B,0} = 1/2$ while $\sigma_{B,1} = 1$. It then follows that

$$KS(\mathcal{B}_\tau^{t_b}, (0,0)) = \{(w,w) : w = 1 - w^{\sigma_\tau}\}.$$

For the case $t_b = 0, \tau = 0$ this implies that the KS-solution corresponds to the chances of receiving R shared as $(\frac{3-\sqrt{5}}{2}, \frac{\sqrt{5}-1}{2})$. If $t_b = 1, \tau = 1$, because both individuals have EU-preferences in this period, their outcome is similar to the preceding case but discounted.

Now considering the case $t_b = 0, \tau = 1$. The KS-solution gives probabilities of obtaining the rent $(1/2, 1/2)$. As the payment is delayed, the corresponding utilities are $(D_A(1)/2, D_B(1)/2)$. Individual A compares utility relative to the bargaining outcome time being $\tau = 0$, thus $\frac{D_A(1)}{2}$ with $\frac{3-\sqrt{5}}{2}$. therefore, individual A would prefer to delay the bargaining outcome if $D_A(1) > 3 - \sqrt{5} \approx 0.76$.

Let us explain the intution for why delayed bargaining outcomes can be beneficial. In our example, individual A's probability of receiving the rent is determined such that $p = 1 - p^{\sigma_{B,\tau}}$. By implicit differentiation we obtain

$$\frac{dp_A}{d\sigma_{B,\tau}} = -\frac{ln(p_A)p^{\sigma_{B,\tau}}}{1 + \sigma_{B,\tau}p^{\sigma_{B,\tau}-1}},$$

which is positive for all $p \in (0,1)$. This means that individual A's chance of receiving the rent increases with individual B's risk tolerance, provided that $\sigma_{B,t_b} < 1$. The argument used here follows the explanation provided by Köbberling and Peters (2003): the lower risk tolerance of B corresponds to more pessimistic behavior, which means that individual B is more difficult to please. Hence, A must offer B a better deal at A's own expense. With increasing risk tolerance for the bargaining outcome, B is easier to please, thus A's chances of obtaining the rent improve. Provided that this improvement is large enough to counteract the fact that delayed outcomes are discounted, individual A will prefer the bargaining outcome delay.

5 Conclusions

Motivated by empirical observations that a joint delay in resolution and receipt of risky outcomes seems to generate more risk tolerance, we have a proposed a setup and a parametric model that can explain postitive and negative effects of delay with a single index. For this purpose we adopted prospect theory, the most successful descriptive theory for risk, within a temporal setting in which only attitudes towards probabilities can vary with delayed prospects. Specifically, the CRS-probability weighting function of Abdellaoui, et al. (2010) was considered, which identifies a single parameter as index for sensitivity towards changes in probabilities.

To accommodate the empirical findings on risk tolerance we have developed preference conditions that describe such choice behavior. Subsequently, we have demonstrated that, within our framework, there is a one-to-one correspondence between increased sensitivity with delay as a choice behavior, and revealed risk tolerance with delay. Our Theorem 1 also provides a relationship of these choice behaviors to the sensitivity parameters of the CRS-weighting functions at different points in time. Our application to cooperative bargaining indicates that delays can be beneficial for individuals bargaining over indivisible objects if opponents display increasing risk tolerance. This is a simple application of a model that can be regarded as a minor deviation from discounted expected utility, where "minor" is meant to reflect that empirical regularities have been incorporated yet a tractable version of a potentially much more general model has been obtained.

Appendix: Proofs

Proof of Proposition 1: As mentioned in the main text, the proof of this proposition follows from the results of Köbberling and Wakker (2003). Our time invariance of outcome tradeoffs comes down to Köbberling and Wakker's tradeoff consistency property, which they use to obtain a subjective expected utility foundation (see their Theorem 5). We assume that general DPT holds, which, together with the time invariance of outcome tradeoffs, implies that the conditions in statement (ii) of Köbberling and Wakker's Theorem 5 are satisfied. As indicated in the main text, this implies that utility in DPT is time independent. Given that our general DPT model satisfies a strong outcome monotonicity property, all periods are essential for the preference; hence, the uniqueness results stated in Köbberling and Wakker's Observation 6(c) apply. Different to the normalization employed for obtaining uniqueness of subjective probabilities in subjective expected utility derivations, the normalization used for the unique weights in DPT each time-period is such that $D(0) = 1$ and, due to impatience, the weights are strictly decreasing. This completes the proof of Proposition 1. □

Proof of Proposition 2: We can assume that Proposition 1 holds. That is, the preference is represented by DPT with common time independent utility $u(\cdot)$, general discount function $D(\cdot)$, and time-dependent CRS probability weighting function $w_t(\cdot)$. Next, we show that $\succcurlyeq$ on $\mathcal{P}$ satisfies time-invariant propensity to gamble if and only if $\eta := \eta_0 = \eta_t$ for all time periods $t \in \{1, \ldots T\}$. Recall that the preference relation $\succcurlyeq$ on $\mathcal{P}$ satisfies time-invariant propensity to gamble if for all time periods $t, s \in \{0, \ldots, T\}$, $s < t$, all outcomes $a, b, x, y \in \mathbb{R}_+$ and all profiles $\mathbf{x} \in \mathcal{P}$ the following holds:

$$(\eta_s : a, 1 - \eta_s : x)_s\mathbf{x} \sim (\eta_s : b, 1 - \eta_s : y)_s\mathbf{x} \Rightarrow (\eta_t : a, 1 - \eta_t : x)_t\mathbf{x} \sim (\eta_t : b, 1 - \eta_t : y)_t\mathbf{x}.$$

Obviously, if $\eta := \eta_0 = \eta_t$ for all time periods $t \in \{1, \ldots T\}$, then time-invariant propensity to gamble follows immediately.

Next, assume that time-invariant propensity to gamble holds. Set $\eta := \eta_0$ and let $a, b \in \mathbb{R}_+$ be arbitrary distinct outcomes and $\mathbf{x} \in \mathcal{P}$ be an arbitrary profile. By continuity of u there exist distinct outcomes $x, y \in \mathbb{R}_+$ such that $(\eta : a, 1-\eta : x)_0\mathbf{x} \sim (\eta : b, 1-\eta : y)_0\mathbf{x}$. Substitution of DPT as given by Proposition 1 implies

$$D(0)PT_0(\eta : a, 1-\eta : x) + \sum_{t=1}^{T} D(t)PT_t(\tilde{x}_t) = D(0)PT_0(\eta : b, 1-\eta : y) + \sum_{t=1}^{T} D(t)PT_t(\tilde{x}_t),$$

which, after cancellation of common terms, gives:

$$w_0(\eta)u(a) + [1 - w_0(\eta)]u(x) = w_0(\eta)u(b) + [1 - w_0(\eta)]u(y)$$

or, using the property of the CRS weighting functions that $w_0(\eta) = \eta$ and our assumption that $0 < \eta < 1$, equivalently

$$\frac{\eta}{1-\eta} = \frac{u(y) - u(x)}{u(a) - u(b)}. \tag{5}$$

Further, time-invariant propensity to gamble implies that $(\eta_s : a, 1-\eta_s : x)_s\mathbf{x} \sim (\eta_s : b, 1-\eta_s : y)_s\mathbf{x}$ holds for all time periods $s \in \{1, \ldots, T\}$. This implies, by a similar argument as above, that

$$\frac{\eta_s}{1-\eta_s} = \frac{u(y) - u(x)}{u(a) - u(b)}.$$

Given Equation (5) and the fact that $g(\eta) = \eta/(1-\eta)$ is strictly increasing on the interval $(0, 1)$, it follows that $\eta_s = \eta$ for all $s \in \{1, \ldots, T\}$. Hence, $w_s(\eta) = \eta$ for all time periods $s \in \{0, \ldots, T\}$. This completes the proof of Proposition 2. □

Proof of Theorem 1: We assume that the preference $\succcurlyeq$ on $\mathcal{P}$ is represented by DPT^η as in Proposition 2. Next we assume Statement (i) and derive Statement (iii) of the theorem. If $\succcurlyeq$ on $\mathcal{P}$ satisfies increasing sensitivity with delay, then for all outcomes $x > y > 0$ such that $u(x) - u(y) = u(y) - u(0)$, and all $\mathbf{z} \in \mathcal{P}$ such that for some time period $t \in \{0, \ldots, T-1\}$ and probabilities p_t, p_{t+1} the two indifferences

$$(\eta : x, 1-\eta : y)_t\mathbf{z} \sim (p_t : x, 1-p_t : 0)_t\mathbf{z}$$

and

$$(\eta : x, 1-\eta : y)_{t+1}\mathbf{z} \sim (p_{t+1} : x, 1-p_{t+1} : 0)_{t+1}\mathbf{z}$$

are satisfied, it follows that

$$p_t > p_{t+1}.$$

As continuity of u ensures that we can always find outcomes $x > y > 0$ such that $u(x) - u(y) = u(y) - u(0)$ and continuity (and strict monotonicity) of the probability weighting functions ensures that there exists (unique) probabilities $p_t, p_{t+1} \in (\eta, 1)$ such that the above indifferences hold, substitution of of DPT^{η} implies that

$$\eta u(x) + (1-\eta)u(y) = w_t(p_t)u(x) + [1 - w_t(p_t)]u(0)$$

and

$$\eta u(x) + (1-\eta)u(y) = w_{t+1}(p_{t+1})u(x) + [1 - w_{t+1}(p_{t+1})]u(0)$$

from which

$$w_t(p_t) = w_{t+1}(p_{t+1})$$

follows for all $t \in \{0, \ldots, T-1\}$. Further, exploiting that $u(x) - u(y) = u(y) - u(0)$ we find that

$$\begin{aligned} \eta u(x) + (1-\eta)u(y) &= w(p_t)u(x) + [1 - w_t(p_t)]u(0) \\ \Leftrightarrow \eta[u(x) - u(y)] + u(y) &= w_t(p_t)[u(x) - u(0)] + u(0) \\ \Leftrightarrow (\eta + 1)[u(x) - u(y)] &= 2w_t(p_t)[u(x) - u(y)], \end{aligned}$$

from which we obtain that

$$w_t(p_t) = \frac{\eta + 1}{2} \text{ for all } t \in \{0, \ldots, T\}.$$

Next we substitute the CRS weighting function in the preceding equation and obtain

$$\begin{aligned} 1 - (1-\eta)^{1-\sigma_t}(1-p_t)^{\sigma_t} &= (\eta + 1)/2 \\ \Leftrightarrow 1 - (\eta + 1)/2 &= (1-\eta)^{1-\sigma_t}(1-p_t)^{\sigma_t} \\ \Leftrightarrow 1/2 &= [(1-p_t)/(1-\eta)]^{\sigma_t} \\ \Leftrightarrow (1-\eta)(1/2)^{1/\sigma_t} &= 1 - p_t \\ \Leftrightarrow 1 - (1-\eta)(1/2)^{1/\sigma_t} &= p_t \text{ for all } t \in \{0, \ldots, T\}. \end{aligned}$$

Now we use the condition that $p_t > p_{t+1}$ for all $t \in \{0, \ldots, T-1\}$ together with the preceding equation to find that

$$1 - (1-\eta)(1/2)^{1/\sigma_t} = p_t > p_{t+1} = 1 - (1-\eta)(1/2)^{1/\sigma_{t+1}}$$

which implies

$$(1/2)^{1/\sigma_t} < (1/2)^{1/\sigma_{t+1}} \Leftrightarrow 1/\sigma_t > 1/\sigma_{t+1}$$

for all $t \in \{0, \ldots, T-1\}$. As the parameters σ_s are strictly bounded between 0 and 1 for all time periods s, we obtain $\sigma_t < \sigma_{t+1}$ for all $t \in \{0, \ldots, T-1\}$. This completes the derivation of Statement (iii) from Statement (i).

Next we assume Statement (iii) and prove Statement (ii). Assume an arbitrary time period $t \in \{0, \ldots, T-1\}$, outcomes $x > y > 0$, profile $\mathbf{z} \in \mathcal{P}$, and probabilities $\eta < p < 1$ such that

$$(p : x, 1-p : 0)_t\mathbf{z} \sim (1 : y)_t\mathbf{z}.$$

We need to show that

$$(p : x, 1-p : 0)_{t+1}\mathbf{z} \succ (1 : y)_{t+1}\mathbf{z}.$$

Assume to the contrary that

$$(p : x, 1-p : 0)_{t+1}\mathbf{z} \preccurlyeq (1 : y)_{t+1}\mathbf{z}.$$

From the indifference $(p : x, 1-p : 0)_t\mathbf{z} \sim (1 : y)_t\mathbf{z}$ and substitution of DPT^{η} it follows that

$$w_t(p)[u(x) - u(0)] = u(y) - u(0)$$

while the preference $(p : x, 1-p : 0)_{t+1}\mathbf{z} \preccurlyeq (1 : y)_{t+1}\mathbf{z}$ and substitution of DPT^{η} implies

$$w_{t+1}(p)[u(x) - u(0)] \leq u(y) - u(0).$$

Combining the latter inequality with the preceding equation and cancellation of common terms gives

$$w_{t+1}(p) \leq w_t(p)$$

But this contradicts the fact that, as a result of $\sigma_{t+1} > \sigma_t$, it must be that the CRS weighting function w_{t+1} gives higher values to probabilities in $(\eta, 1)$ than w_t does. Hence, $(p : x, 1-p : 0)_{t+1}\mathbf{z} \succ (1 : y)_{t+1}\mathbf{z}$ follows. As $t \in \{0, \ldots, T-1\}$, outcomes $x > y > 0$, profile $\mathbf{z} \in \mathcal{P}$, and probabilities $\eta < p < 1$ were arbitrary chosen, the implication holds for all $t \in \{0, \ldots, T-1\}$, outcomes $x > y > 0$, profile $\mathbf{z} \in \mathcal{P}$, and probabilities $\eta < p < 1$. This means that the preference $\succcurlyeq$ satisfies increasing risk tolerance with delay, which completes the derivation of Statement (ii) from Statement (iii).

Next we assume that Statement (ii) holds and derive Statement (i). Suppose that for some arbitrary outcomes $x > y > 0$ such that $u(x) - u(y) = u(y) - u(0)$, profile $\mathbf{z} \in \mathcal{P}$, time period $t \in \{0, \ldots, T-1\}$ and probabilities p_t, p_{t+1} we have the two indifferences

$$(\eta : x, 1-\eta : y)_t\mathbf{z} \sim (p_t : x, 1-p_t : 0)_t\mathbf{z}$$

and

$$(\eta : x, 1-\eta : y)_{t+1}\mathbf{z} \sim (p_{t+1} : x, 1-p_{t+1} : 0)_{t+1}\mathbf{z}.$$

We need to show that $p_t > p_{t+1}$. By continuity and strict monotonicity of the utility u there exists some outcome $0 < z < x$ such that

$$(p_t : x, 1-p_t : 0)_t\mathbf{z} \sim (1 : z)_t\mathbf{z}.$$

Now increasing risk tolerance with delay implies that

$$(p_t : x, 1 - p_t : 0)_{t+1}\mathbf{z} \succ (1 : z)_{t+1}\mathbf{z}.$$

Continuity and strict monotonicity of the CRS-probability weighting function w_{t+1} implies that there exists a positive probability $q < p_t$ such that

$$(q : x, 1 - q : 0)_{t+1}\mathbf{z} \sim (1 : z)_{t+1}\mathbf{z}.$$

Further, by transitivity $(p_t : x, 1 - p_t : 0)_t\mathbf{z} \sim (1 : z)_t\mathbf{z}$ and $(\eta : x, 1 - \eta : y)_t\mathbf{z} \sim (p_t : x, 1 - p_t : 0)_t\mathbf{z}$ imply

$$(\eta : x, 1 - \eta : y)_t\mathbf{z} \sim (1 : z)_t\mathbf{z},$$

such that substitution of DPT^η and cancellation of common terms gives

$$\eta u(x) + (1 - \eta)u(y) = u(z).$$

This equation is independent of the time period t, hence, using the fact that η is a common fix point of all CRS-weighting functions, we obtain $w_{t+1}(\eta)u(x) + [1 - w_{t+1}(\eta)]u(y) = u(z)$ from which we derive

$$(\eta : x, 1 - \eta : y)_{t+1}\mathbf{z} \sim (1 : z)_{t+1}\mathbf{z}.$$

Given our assumption that $(\eta : x, 1 - \eta : y)_{t+1}\mathbf{z} \sim (p_{t+1} : x, 1 - p_{t+1} : 0)_{t+1}\mathbf{z}$ and transitivity, we obtain

$$(p_{t+1} : x, 1 - p_{t+1} : 0)_{t+1}\mathbf{z} \sim (1 : z)_{t+1}\mathbf{z}.$$

Now increasing risk tolerance with delay implies that

$$(p_{t+1} : x, 1 - p_{t+1} : 0)_t\mathbf{z} \prec (1 : z)_t\mathbf{z}.$$

Together with $(p_t : x, 1 - p_t : 0)_t\mathbf{z} \sim (1 : z)_t\mathbf{z}$ and transitivity we obtain

$$(p_{t+1} : x, 1 - p_{t+1} : 0)_t\mathbf{z} \prec (p_t : x, 1 - p_t : 0)_t\mathbf{z},$$

which can only hold if $p_{t+1} < p_t$ (as the CRS-weighting functions are strictly increasing).

This conclusion has been obtained starting with arbitrary outcomes $x > y > 0$ such that $u(x) - u(y) = u(y) - u(0)$, arbitrary profile $\mathbf{z} \in \mathcal{P}$ and time period $t \in \{0, \ldots, T-1\}$ such that the indifferences

$$(\eta : x, 1 - \eta : y)_t\mathbf{z} \sim (p_t : x, 1 - p_t : 0)_t\mathbf{z}$$

and

$$(\eta : x, 1-\eta : y)_{t+1}\mathbf{z} \sim (p_{t+1} : x, 1-p_{t+1} : 0)_{t+1}\mathbf{z}$$

hold. Hence, the conclusions is valid for all probability midpoints p_t, p_{t+1} and time periods $t \in \{0, \ldots, T-1\}$. This completes the derivation of Statement (i) from Statement (ii).

To summarize: First, we have shown that Statement (i) implies Statement (iii), then we have derived Statement (ii) from Statement (iii) and , finally, we have proven that Statement (ii) implies Statement (i). This completes the proof of the theorem. □

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